

A BROWN DWARF COMPANION FOR THE ACCRETING MILLISECOND PULSAR SAX J1808.4–3658

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ABSTRACT

The *BeppoSAX* Wide Field Cameras have revealed a population of faint neutron star X-ray transients in the Galactic bulge. King conjectured that these neutron stars are accreting from brown dwarfs with a time-averaged mass transfer rate $\langle \dot{M} \rangle \approx 10^{-11} M_{\odot} \text{ yr}^{-1}$ that is low enough for accretion disk instabilities. We show that the measured orbital parameters of the 401 Hz accreting millisecond pulsar SAX J1808.4–3658 support this hypothesis. A main-sequence mass donor requires a nearly face-on inclination and a higher $\langle \dot{M} \rangle$ than observed, and can thus be excluded. However, the range of allowed inclinations is substantially relaxed and the predicted $\langle \dot{M} \rangle$ is consistent with that observed if a hot $0.05 M_{\odot}$ brown dwarf is the donor.

The remaining puzzle is explaining the brown dwarf radius required ($0.13 R_{\odot}$) to fill the Roche lobe. Recent observational and theoretical work has shown that all transiently accreting neutron stars have a minimum luminosity in quiescence set by the time-averaged mass transfer rate onto the neutron star. We show here that the constant heating of the brown dwarf by this quiescent neutron star emission appears adequate to maintain the higher entropy implied by a $0.13 R_{\odot}$ radius. All of our considerations very strongly bolster the case that SAX J1808.4–3658 is a progenitor to compact millisecond radio pulsar binaries (e.g. like those found by Camilo and collaborators in 47 Tuc). The very low $\langle \dot{M} \rangle$ of SAX J1808.4–3658 implies that the progenitors to these radio pulsars are long-lived ($\sim \text{Gyr}$) transient systems, rather than short-lived ($\sim \text{Myr}$) Eddington-limited accretors. Hence, the accreting progenitor population to millisecond radio pulsars in 47 Tuc could still be present and found in quiescence with *Chandra*.

Subject headings: binaries: close — pulsars: general — pulsars: individual: SAX J1808.4–3658 — stars: low-mass — stars: neutron — x-rays: binaries

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1. INTRODUCTION

X-ray monitoring of the Galactic bulge with the *BeppoSAX* Wide Field Cameras (WFCs) has identified seven extremely low-luminosity, transient X-ray sources (Heise et al. 1998). All show type I X-ray bursts from unstable thermonuclear burning, indicating that they are accreting neutron stars (NSs) with weak ($\ll 10^{10}$ G) surface magnetic field strengths. Their peak fluxes (apart from the type I bursts) are below 100 mCrab, corresponding to a luminosity of $< 5 \times 10^{36} \text{ erg s}^{-1}$ at the Galactic center. At least three recur, with week-long outbursts coming at intervals ranging from months to years. King (2000) has shown that these sources form a distinct class of NSs in low-mass X-ray binaries (LMXBs). Their unusually low time-averaged mass transfer rates ($\langle \dot{M} \rangle \sim 10^{-11} M_{\odot} \text{ yr}^{-1}$) allow for accretion disk instabilities to operate and cause the recurrent outbursts. Such low $\langle \dot{M} \rangle$'s occur when orbital angular momentum is lost via gravitational radiation (GR) and when the donor mass is in the brown dwarf (BD) regime ($\lesssim 0.08 M_{\odot}$; Verbunt & van den Heuvel 1995, King 2000). This is the natural endpoint of mass transfer evolution for a low-mass donor (e.g. Rappaport, Verbunt & Joss 1983).

A famous member of this class is SAX J1808.4–3658, the

only known accretion-powered millisecond pulsar. It was first detected as a X-ray transient with type I X-ray bursts in 1996 September by the WFCs in a 20 day outburst (in 't Zand et al. 1998). A recent reanalysis of these data has yielded a distance of 2.5 kpc from the photospheric radius expansion observed during the type I bursts (in 't Zand et al. 2001). During a second outburst in 1998 April, observations with the *Ross X-Ray Timing Explorer* (RXTE) revealed persistent 401 Hz X-ray pulsations (Wijnands & van der Klis 1998) and 2 hr binary motion (Chakrabarty & Morgan 1998, hereafter CM98).

The measured orbital parameters of this system allow us to solve for the binary companion's mass, M_c , and radius, R_c , as a function of orbital inclination. White dwarf companions are ruled out while low-mass hydrogen main sequence companions are allowed only for highly improbable inclination values and an $\langle \dot{M} \rangle$ inconsistent with the observations (CM98). In this paper, we show that a more consistent solution involves a $M_c \approx 0.05 M_{\odot}$ BD as the mass donor. This substantially relaxes the inclination to more likely values and gives an $\langle \dot{M} \rangle$ that agrees with observations. The puzzle is why such a low-mass donor would fill the Roche lobe at an orbital period of 2 hours.

We conjecture here that the larger entropy implied by the $R_c \approx 0.13 R_{\odot}$ radius can be maintained by the continuous heat-

ing of the BD in quiescence by the thermal emission from the NS. This naturally makes this system a progenitor to the fast millisecond radio pulsars (MRPs) in short (< 6 hr) orbits and shows that there is a \sim Gyr phase of accretion at very low rates. This is in contrast to progenitor scenarios (e.g. Ruderman, Shaham and Tavani 1989) which give \dot{M} 's at least two orders of magnitude higher.

2. CONSTRAINTS ON THE BINARY INCLINATION

The binary parameters of SAX J1808.4–3658 were precisely measured, with orbital period $P_{\text{orb}} = 2.01$ hr and a projected radius $a_x \sin i = 62.8$ light-ms (CM98). The mass function is

$$f_x \equiv \frac{(M_c \sin i)^3}{(M_x + M_c)^2} = \frac{4\pi^2 (a_x \sin i)^3}{GP_{\text{orb}}^2} = 3.8 \times 10^{-5} M_\odot, \quad (1)$$

where i is the binary inclination (the angle between the line of sight and the orbital angular momentum vector) and M_x is the NS mass. Given M_x and i , equation (1) yields M_c , as shown in Figure 1 for $M_x = 1.4 M_\odot$ and $M_x = 2 M_\odot$. The minimum companion mass ($i = 90^\circ$) is then $\simeq 0.043$ (0.054) M_\odot for $M_x = 1.4$ (2.0) M_\odot , far below the hydrogen burning limit of $\approx 0.08 M_\odot$. For randomly selected binaries, the a priori probability of observing a system with $i < i_0$ is $(1 - \cos i_0)$, so the likeliest solutions are those with BD donors.

Mass transfer from the Roche-lobe-filling¹ low mass companion is driven by angular momentum loss from GR, \dot{J}_{GR} (see review by Verbunt & van den Heuvel 1995). The mass transfer rate is then $\dot{M}_{\text{GR}} = 3M_c \dot{J}_{\text{GR}} / 2J$, or

$$\dot{M}_{\text{GR}} \approx 3.8 \times 10^{-11} M_\odot \text{ yr}^{-1} \left(\frac{M_c}{0.1 M_\odot} \right)^2 \left(\frac{M_x}{1.4 M_\odot} \right)^{2/3} \left(\frac{P_{\text{orb}}}{2 \text{ hr}} \right)^{-8/3}, \quad (2)$$

where we have set² $n = -1/3$ in the donor's mass-radius relation, $R_c \propto M_c^n$. This is plotted in the bottom panel of Figure 1. The mass transfer rate in equation (2) is smaller by a factor of two for a main sequence mass-radius relation ($n = 1$).

We now compare this to the measured $\langle \dot{M} \rangle$. The 1998 April outburst of SAX J1808.4–3658 was monitored by a series of pointed *RXTE* observations, from which a total fluence of $\approx 4.3 \times 10^{-3}$ erg cm⁻² (3–150 keV; Gilfanov et al. 1998) was measured. The recurrence time of the outbursts is roughly 20 months (in 't Zand et al. 1998; Marshall 1998; van der Klis et al. 2000). Presuming that the 1998 April outburst was typical, the time-averaged flux is 9×10^{-11} erg cm⁻² s⁻¹, giving

$$\langle \dot{M} \rangle \approx 5 \times 10^{-12} M_\odot \text{ yr}^{-1} \left(\frac{d}{2.5 \text{ kpc}} \right)^2 \left(\frac{M_x}{1.4 M_\odot} \right)^{-1}, \quad (3)$$

which is most consistent for $M_c \approx 0.05 M_\odot$, thus implying that the binary is not viewed face-on.

Three other pieces of evidence also suggest that SAX J1808.4–3658 is not viewed face-on. First, there is a 2% modulation in X-ray intensity at the orbital period, with the minimum occurring when the NS is behind the companion (CM98; Lee, Chakrabarty, & Morgan 2001, in preparation). This suggests that i must be large enough to allow partial X-ray blockage from some circumbinary material. Second, the 2 hour single-peaked modulation in the optical intensity (Giles, Hill,

& Greenhill 1999) has been confirmed by Homer et al. (2001). Flux minimum occurs when the NS is behind the companion, suggesting that X-ray heating of the companion is the origin of the modulation; again implying inclinations other than face-on. Finally, detailed modeling of the optical companion's multi-band photometry during outburst with a simple X-ray heated disk model suggests that $\cos i < 0.45$ (Wang et al. 2001, in preparation). Though clearly not a face-on system, the absence of deep X-ray eclipses indicates that we must be viewing SAX J1808.4–3658 with $\cos i > 0.15$ (CM98).

In summary, a consideration of the mass transfer rates and inclination constraints lead us to conclude that the likely Roche lobe filling donor has a mass $M_c \approx 0.05 M_\odot$. We now discuss how such a system is formed and maintains Roche-Lobe filling at the two hour orbital period.

3. COMPANION RADIUS AND BINARY EVOLUTION

The mean density of a Roche-lobe-filling companion (for the case where $M_c \ll M_x$) is set by the binary period (Faulkner, Flannery, & Warner 1972), so that $R_c = 0.17 (M_c / 0.1 M_\odot)^{1/3} R_\odot$, as shown by the dark solid curve in Figure 2. Also plotted are the mass-radius relations for cold helium white dwarfs (Paczynski 1967), low-mass hydrogen main sequence stars (Tout et al. 1996), and BDs of ages 0.1, 0.5, 1.0 and 5.0 Gyr (Chabrier et al. 2000). Note that the mass-radius relation for any Roche-lobe-filling solution allowed by the observations must intersect (or at least lie near) the dark solid curve in Figure 2. Thus, we see immediately that cold helium white dwarfs are too small (for any mass). Low-mass hydrogen main sequence stars give a solution at $M_c = 0.17 M_\odot$, corresponding to $\cos i = 0.95$. This solution has a small a priori probability (5%); moreover, it would give a mass transfer rate (from equation (2) with $n = 1$) of $\langle \dot{M} \rangle > 5 \times 10^{-11} M_\odot \text{ yr}^{-1}$, a factor of ten higher than observed. Thus, the solution we consider most likely is a $0.05 M_\odot$ BD with radius $R_c \approx 0.13 R_\odot$.

Many (e.g. Paczynski & Sienkiewicz 1981; Rappaport, Verbunt & Joss 1983) have shown that the mass transfer evolution (under GR losses) of a $M < M_\odot$ main sequence donor leads to a BD companion within a few Gyrs. The initial evolution of these systems is towards shorter orbital periods when the Kelvin-Helmholtz (KH) time is shorter than the mass transfer time scale (this is typically when the donor is on the main sequence). Once the KH time becomes longer than the mass transfer time scale, the donor expands under further mass loss (if adiabatic, then $n = -1/3$), and the period increases. This sets the minimum orbital period at ≈ 80 minutes and if the BD is allowed to cool once nuclear burning has ceased, the companion would have a mass $\ll 0.05 M_\odot$ when a period of 2 hours is reached (e.g. Rappaport et al. 1983). Hence, some inhibition of the contraction must occur in SAX J1808.4–3658.

The issue of expanding BD companions by external heating (or driving winds) came to a sharp focus with the discovery of the eclipsing optical counterpart to PSR 1957+20 (Fruchter et al. 1990). Many models were constructed, some of which relied on tidal heating of a low-mass, asynchronous object for the expansion (e.g. Applegate and Shaham 1994). Rasio, Pfahl, and Rappaport (2000) present a similar scenario for the MRPs in 47 Tuc in short orbital periods, implying mass transfer rates a

¹Most LMXBs accrete via Roche lobe overflow. Substantial mass loss through an intrinsic stellar wind is unlikely for very low-mass donors.

²Equation (2) agrees with King, Kolb and Szuszkiewicz (1997) but is a factor of three larger than equation (3) of King (2000). Though we are not certain, it appears that King's number is lower because he presumed $n = 1$ for his mass-radius relation and dropped a significant figure. Our choice of $n = -1/3$ is for the isentropic and heated BD companion motivated in §'s 3 and 4.

factor of 100-1000 higher than observed in SAX J1808.4–3658. Ruderman et al. (1989) had previously considered the possibility that the light companion was heated at a rate adequate to drive high mass (and angular momentum) loss that would increase the mass transfer rate in the binary to near-Eddington limits (see Bhattacharya 1995 for a review). Some of their motivation was the lack of observed low-accretion rate LMXB's. The *BeppoSAX* discoveries have clearly changed the situation and motivated us to reconsider GR-driven systems at late times when the donor mass is below the hydrogen burning limit.

4. THE THERMOSTAT OF X-RAY HEATING IN QUIESCENCE

The companion is heated to $> 2 \times 10^4$ K during the outbursts in SAX J1808.4–3658. We presume that this transient external heating has little effect on the interior of the BD; rather we focus on the continuous heating in quiescence from the hot NS. Bildsten and Brown (1997) showed that the quiescent heating of the outer disk was insufficient to suppress the disk instability responsible for the outbursts. However, it appears sufficient to dramatically alter the entropy evolution of the low-mass stellar companion once nuclear burning has ceased.

Transiently accreting NSs are always detected in quiescence at an X-ray luminosity $> 10^{32}$ erg s $^{-1}$ (see Bildsten and Rutledge 2000 for an overview). This emission has two components: a soft thermal component from the NS surface (Brown, Bildsten & Rutledge 1998; Rutledge et al. 1999, 2000, 2001) and a hard power-law tail of unknown origin (see Campana & Stella 2000 for a discussion). The thermal emission level is well predicted by the relation of Brown et al. (1998)

$$L_{\text{th}} \approx 6 \times 10^{32} \text{ erg s}^{-1} \left(\frac{\langle \dot{M} \rangle}{10^{-11} M_{\odot} \text{ yr}^{-1}} \right), \quad (4)$$

which arises from the deep nuclear heating that the NS receives every outburst.

For the time-averaged flux derived above, equation (4) predicts thermal emission at 4.5×10^{-13} erg cm $^{-2}$ s $^{-1}$. There are two weak detections of the quiescent X-ray emission from SAX J1808.4–3658. Stella et al. (2000) report $(1-2) \times 10^{-13}$ erg cm $^{-2}$ s $^{-1}$ (0.5–10 keV) with *BeppoSAX* (in excellent agreement once the spectral corrections are applied; see Table 4 of Rutledge et al. 2000) while Dotani, Asai & Wijnands (2000) report a value about a factor of 3 lower from *ASCA*.

We use the quiescent luminosity from equation (4) as a lower-limit to the heating and find the temperature, T_{H} , of the companion on the side facing the NS, $\sigma_{\text{SB}} T_{\text{H}}^4 = L_{\text{th}}/4\pi(a-R)^2$, as a function of $\langle \dot{M} \rangle$. For plausible inclinations of SAX J1808.4–3658, $T_{\text{H}} \approx 3800-4800$ K, far above the expected value for a BD that has cooled after passing through the period minimum. We conjecture that this continuous heating fixes the entropy of the BD at a value higher than would occur in the absence of heating, where the entropy is free to decrease once nuclear burning has halted. This would also increase the minimum orbital period of such binaries.

There are no appropriate calculations that allow us to accurately estimate the slowing of contraction from partial irradiation at these levels. However, the effects of less irradiation of smaller mass objects, the irradiated extrasolar giant planets, lead to radii 50% larger than expected (Burrows et al. 2000), as

has been observed for the transiting planet around HD 209458 (Brown et al. 2001).

5. CONCLUSIONS AND COMPARISONS TO MILLISECOND PULSARS

In addition to making the case that the companion to SAX J1808.4–3658 is a brown dwarf, our work also places it in relation to the short orbital period (< 6 hr) fast ($P_s < 5$ ms) MRPs both in the field (PSR J2051–0827, Stappers et al. 1999) and in globular clusters (47 Tuc I, J, O, P and R, Camilo et al. 2000; PSR J1807–2459 in NGC 6544, Ransom et al. 2001; PSR 1908+00 in NGC 6760, Deich et al. 1993). The longer orbital period for PSR 1957+20 places it far outside of this group. The top panel in Figure 3 shows M_c for $\cos i = 0.5$ for the field pulsar PSR J2051–0827 (open triangle) and globular cluster pulsars (solid squares). All have lower mass companions than SAX J1808.4–3658 (asterisk).

We plot two other related quantities in Figure 3. We use the measured orbital parameters and presume that the companion was, in the past, Roche lobe filling at the current orbital period and transferring mass at the rate set by GR losses with $n = -1/3$. The mass transfer rate (shown as a time scale in the bottom panel) and the heated companion temperature, T_{H} (middle panel) are shown for $\cos i = 0.5$. SAX J1808.4–3658 has the most heated companion and nearly the shortest accretion time scale (47 Tuc R is just barely shorter), consistent with it being a progenitor to the pulsars.

If the heating maintains a constant entropy in the BD of SAX J1808.4–3658, further mass transfer will evolve it along the solid line. Since the heating is clearly dropping, we also plot (dashed line) a constant radius evolution ($n = 0$, or decreasing entropy). These clearly bracket many of the observed systems at longer orbital periods. For $n = 0$, it will take 6.6 Gyrs for SAX J1808.4–3658 to reach a system like PSR J2051–0827 ($M_c = 0.03 M_{\odot}$).

Something eventually allows the BD to contract and fall within the Roche lobe, halting mass transfer and allowing the NS to become a MRP. In this respect, it is important that most of the T_{H} 's for the MRPs are comparable to a BD in the contraction phase (Chabrier et al. 2000), implying that the reduced heating at larger orbital periods might well allow the BD to finally fall within the Roche radius.

SAX J1808.4–3658 is clearly a long-lived (\sim Gyr) X-ray source that undergoes mass transfer at a rate of $\approx 10^{-11} M_{\odot} \text{ yr}^{-1}$. If this long-lived system is the progenitor type to the short-orbital-period MRPs in 47 Tuc, then we expect to see faint X-ray transients in 47 Tuc comparable in number to the MRP binaries. The infrequent sampling and long recurrence times would make their detection in outburst unlikely. However, they should easily be detected in quiescence with *Chandra* at the levels implied by equation (4). Discovery of such a plethora of quiescent, short-orbital-period LMXBs would confirm this connection.

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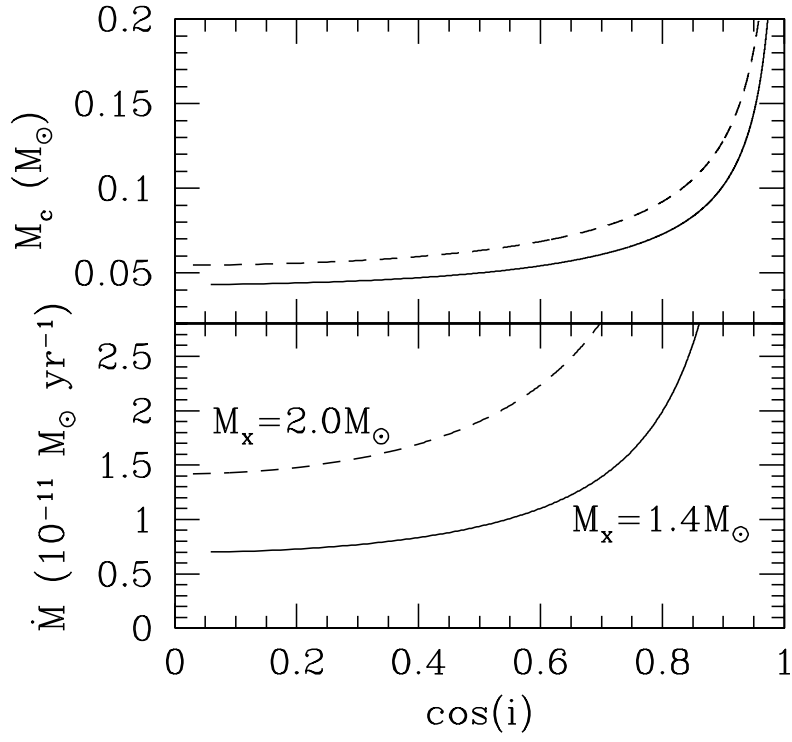


FIG. 1.— The companion mass and mass transfer rate for SAX J1808.4–3658 as a function of inclination angle. The solid (dashed) lines are for a neutron star mass of $1.4(2.0)M_\odot$.

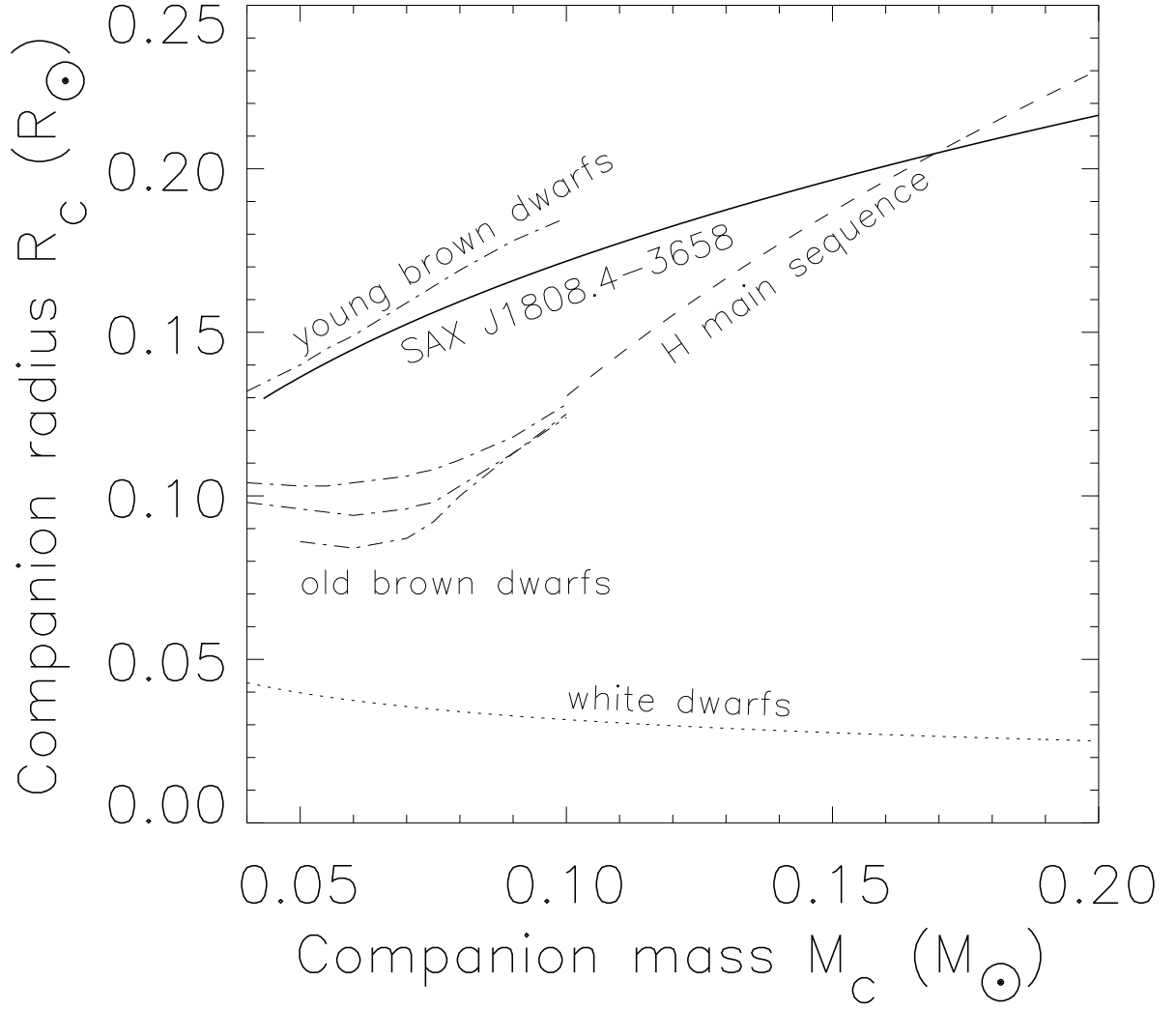


FIG. 2.— Possible companion types for SAX J1808.4–3658. The dark solid curve shows the mass-radius relation for SAX J1808.4–3658 allowed by the observations. The other curves show the theoretical mass-radius relations for cold He white dwarfs (*dotted curve*), low-mass hydrogen mass sequence stars (*dashed curve*), and brown dwarfs of ages (*dot-dash curves*) of 0.1, 0.5, 1.0 and 5.0 Gyr, from top to bottom.

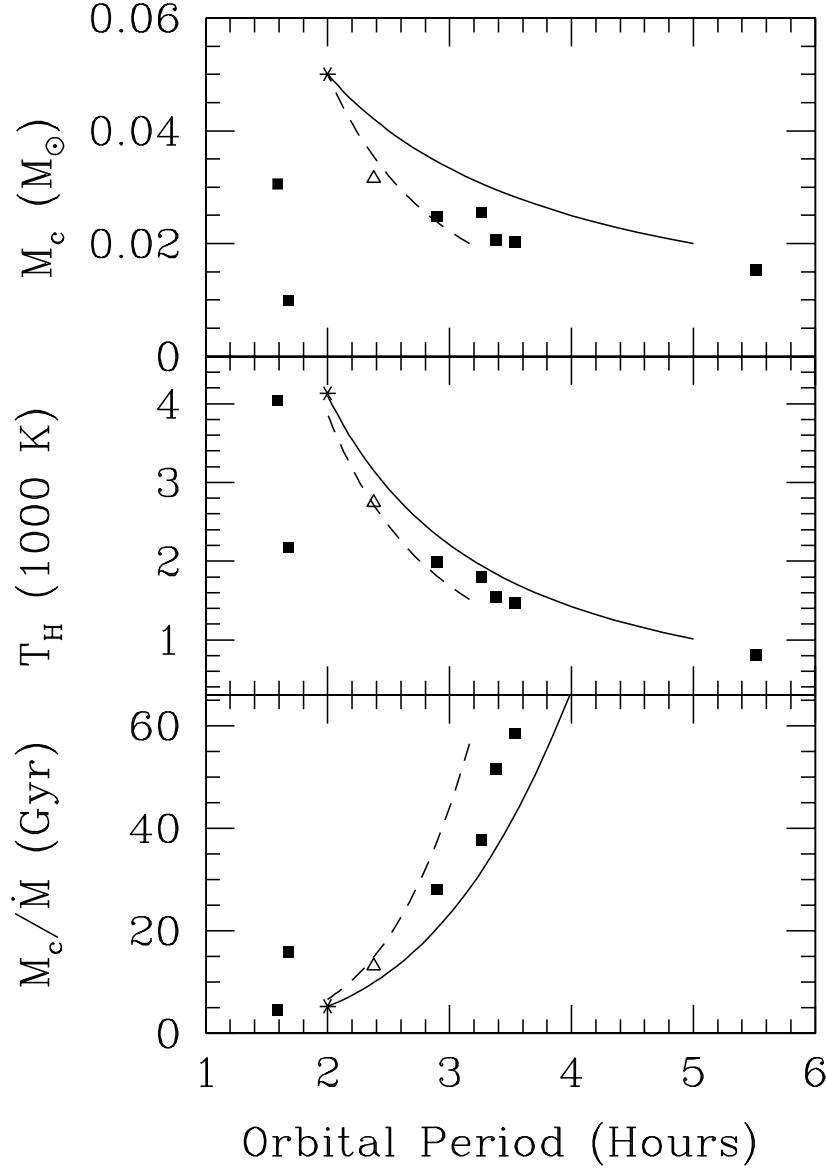


FIG. 3.— Companion masses, heated companion temperature and mass transfer rates for SAX J1808.4–3658 (*asterisk*) and the short orbital period millisecond radio pulsars in the field (PSR J2051-0827, *open triangle*) and in globular clusters (*filled squares*), all presuming a NS mass of $1.4M_\odot$. All points use the measured orbital parameters and assume that $\cos i = 0.5$. The solid (dashed) line denotes the future trajectory for SAX J1808.4–3658 when $n = -1/3$ ($n=0$).